Remarks on our knowledge of modal facts

Alexander Bird
Professor, Department of Philosophy, University of Bristol
E-mail: Alexander.Bird@bristol.ac.uk

Can we have *a posteriori* knowledge of modal facts? And if so, is that knowledge fundamentally *a posteriori*, or does *a priori* intuition provide the modal component of what is known? Though the latter view seems more straightforward, there are also reasons for taking the first option seriously.

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1.

In this paper, I wish to explore the extent to which there can be *a posteriori* knowledge of modal facts, in particular *a posteriori* knowledge that a proposition is necessarily true. (*A posteriori* knowledge that a proposition is possible is, of course, trivial in the cases where we have *a posteriori* knowledge that it is true.) If one assumes, as Kant and many others have done, that knowledge of the truth of propositions that are necessarily true is always *a priori*, then, given the S4 (transitivity) principle $\Box p \rightarrow \Box \Box p$, knowledge that a proposition is necessary is also always *a priori*. As has often been remarked, a distinctive achievement of Saul Kripke and Hilary Putnam (if they are right) has been to drive a wedge between the necessary-contingent and the *a priori–a posteriori* distinctions. The discovery of *a posteriori* necessities is the principal element in this advance. These are exemplified by:

(a) Eric Blair is George Orwell;
(b) Queen Victoria is the mother of Princess Beatrice;
(c) Manganese is the element with atomic number 25;
(d) Water is H$_2$O;
(e) Michelangelo’s *David* is made from marble.

According to the arguments of Kripke, Putnam and others, in each of these cases the proposition in question is true, and is necessarily true.¹ However, the propositions cannot be known to be true *a priori*. There is some disagreement as to the precise nature of the arguments that establish the propositions, especially as regards the degree to which the arguments depend on either semantic premises or metaphysical intuition. Either way, though, the arguments, if successful, establish their conclusions *a priori*. Those arguments proceed by assuming
the propositions above to be true and then showing that they are necessarily true. Thus it is *a priori* for each of the propositions above, that if it is true, then it is necessarily true. If we abbreviate metaphysical necessity with ‘□’ and aprioricity with ‘L’, then for the above propositions the following hold:

(A) □p; ¬Lp; L(p→□p).

Since ¬Lp, we also have ¬L□p. But one can know □p, since one can know *p a posteriori*, and one knows (*a priori*) p→□p. So, our knowledge of □p is *a posteriori*.

The fact that L(p→□p) suggests that there is still some connection remaining between aprioricity and knowledge of necessity. It may appear that although one has *a posteriori* knowledge of □p, that knowledge comes about through the conjoining of *a posteriori* knowledge of a non-modal proposition, p, and *a priori* knowledge of a modal proposition, a proposition concerning necessity or essence, (p→□p). So, it may be argued, knowledge of necessity is still fundamentally *a priori*.

The preceding proposal depends on it being the case that L(p→□p). But we can find cases for which ¬L(p→□p). Elsewhere I have argued that even if the fundamental laws of nature are contingent, some non-fundamental laws may turn out to be necessary.2 Let the higher-level law L concern some substance S. (a) S might (of necessity) require for its existence that some lower-level law C holds — C is required, for example, to enable the internal structure of S to exist. At the same time, (b) L might be made true by C (in the way that in general higher-level laws supervene on and are made true by lower-level laws). If both (a) and (b) hold together, then no possible counterexample to L can exist. Such a counterexample would have to involve S, in which case C holds; in which case L holds also. Take a real example: the law that salt dissolves in water. (a) Salt, being an ionic compound, requires that Coulomb’s law (or something sufficiently like it) should hold. (b) Coulomb’s law (or anything sufficiently like it) will suffice to ensure that salt placed in water will dissolve, since the process of dissolving is one that is brought about by electrical interaction between water and the salt ions. If this argument is correct, then (i) salt dissolves in water is necessarily true. But it is not the case that it is *a priori* that if it is true, it is necessarily true. For the argument just expounded is not an *a priori* one. It depends on knowing certain *a posteriori* truths, such as the ionic nature of salt and the mechanisms of dissolving in water. The *a posteriori* ‘coincidence’ that Coulomb’s law is involved in both is a coincidence that need not be repeated for all other chemical propositions of the same form. Hence, for this proposition:

(B) □p; ¬Lp; ¬L(p→□p).

For those not yet convinced by the preceding argument, there is a quicker and less contentious route to the same conclusion. Consider:

(ii) Eric Blair and George Orwell share the same birthday.

This proposition is necessarily true. There are of course propositions of the same form, ‘X and Y share the same birthday’, that are true, but contingently so. In this case, therefore, there is no inference from the truth of ‘Eric Blair and George Orwell share the same birthday’ to its necessity. The assertions in (B) hold of this proposition also, as they do of the following:
(iii) *water and H₂O both dissolve the same range of beryllium salts.*

It may be remarked that the undecidable propositions of mathematics also provide examples for which (A) is true, even without Kripke-style arguments. For each undecidable proposition, either it or its negation is true. As regards the true one of the pair, it is necessarily true, being a mathematical proposition: $\Box p$. Since it is undecidable, it is not *a priori* that this proposition is true: $\neg \Box p$. But we know *a priori* that any true mathematical proposition is necessarily true: $\Box (p \rightarrow \Box p)$.

Given that (A) does hold for such propositions, we can also generate propositions for which (B) is true. Consider a proposition of the form $s \lor r$ where $s$ is a true undecidable mathematical proposition and $r$ some *a posteriori* contingent proposition. For example:

*(iv) if either the continuum hypothesis is true (s) or Betelgeuse has twelve planets (r) then necessarily either the continuum hypothesis is true or Betelgeuse has twelve planets.*

The proposition $s \lor r$ is necessary if $s$ is true and is contingent if $s$ is false. Since we do not know the truth value of $s$ we do not know whether $s \lor r$ is necessary or contingent. Consequently it is not *a priori* that $(s \lor r) \rightarrow \Box (s \lor r)$.

According to Salmon, Keith Donnellan takes the view that propositions such as: *(v) if atoms of manganese contain 25 protons, then necessarily atoms of manganese contain 25 protons* are also *a posteriori.* The fact that it is the number of protons that is the essence of chemical elementhood is a matter of *a posteriori* discovery. This contrasts with Kripke’s view that such conditionals are *a priori.* If Donnellan was right, then ‘atoms of manganese contain 25 protons’ would express a proposition for which (B) holds. In support of Donnellan’s view, one may consider that it would be possible to understand the claim that an atom had so many protons in its nucleus without appreciating that this was any more significant as regards its essence than the number of neutrons in its nucleus. Salmon himself, however, takes the view that not all principles connecting $p$ and $\Box p$ are *a posteriori.* Salmon’s proposal is that, in general, we can find such *a priori* principles that contain the essentialist knowledge required in generating, along with certain non-modal *a posteriori* propositions, *a posteriori* knowledge of necessary truths. Thus, although there are propositions whose necessity is known *a posteriori,* that knowledge may be factored into *a priori* knowledge of essences plus *a posteriori* knowledge of non-modal propositions. Fundamentally all knowledge of necessity is *a priori.* Salmon’s *a priori* connecting principles would supplement $p$ to give a proposition fulfilling the function of $q$ in the preceding paragraph.

So, Salmon argues that the following conditional principle is *a priori*:

*(S) Given that chemical elements are *in fact* distinguishable by the number of protons in their component atoms, i.e. by their atomic number, every element is such that if it *might* have a certain atomic number $n$, then it *must* have that atomic number $n.$

is *a priori.* This would function as a connecting principle for (iv). However it is not so clear to me that (S) is *a priori.* Let us assume that the law governing the strong nuclear force is contingent (if it is necessary then (B) will hold of it). Imagine a world $w$ where that law differs so that nuclear stability re-
quires that in every atom the number of protons and the number of neutrons were equal. Then the number of neutrons would distinguish the elements. Nevertheless the principle:

\[(S^*) \text{ Given that chemical elements are in fact distinguishable by the number of neutrons in their component atoms, every element is such that if it might have } n \text{ neutrons in its atoms, then it must have } n \text{ neutrons in its atoms.} \]

would be false. We know a posteriori that we do not inhabit \( w \), but we could not know a priori.\(^6\) We do not know a priori that \((S^*)\) is no less rationally preferable than \((S)\). Hence, we cannot know a priori that \((S)\) is true and \((S^*)\) false.

Consider a naturalist who comes across an unfamiliar naturally occurring substance. She investigates its superficial properties sufficiently to identify the substance readily. She gives the substance a name, ‘K’, through the sort of baptismal or other naming procedure countenanced by Kripke and Putnam. What does our naturalist know a priori? Perhaps she knows a priori that:

\[(K1) \text{ If K has constitution C, then necessarily } x \text{ is K only if } x \text{ has constitution C.} \]

However the problem is that her knowledge of the superficial properties need not enable her to know whether the substance she has found is a mineral substance, a non-living organic product, or a living stuff, such as a slime mould. If the last of these is true, \((K1)\) will be false, whereas:

\[(K2) \text{ If K has origin O, then necessarily } x \text{ is K only if } x \text{ has origin O.} \]

will be true. It seems therefore that she cannot know either \((K1)\) or \((K2)\) a priori, but instead must engage in some further a posteriori investigation to work out which she should endorse.

Kripke’s revelation that for some propositions \((A)\) holds, refutes the simplistic claim that knowledge of necessities is always a priori. However even in such cases the fact that \(L(p \rightarrow \square p)\) is true and plays a part in explaining our knowledge of \( \square p \) makes plausible the thought that knowledge of necessity is fundamentally a priori. Even in cases where we do have a posteriori knowledge of \( \square p \), that knowledge can be factored into a posteriori knowledge of the non-modal \( p \) and a priori knowledge of the modal \( p \rightarrow \square p \). The interest of propositions for which \((B)\) holds is, therefore, that this factoring cannot be done straightforwardly. Nonetheless, the defender of the fundamental aprioricity of knowledge of necessity might reply that even in these instances it remains the case that there is some non-modal proposition \( q \) such that \( L(q \square p) \). As regards \((ii) q \) is the proposition that Eric Blair is George Orwell; likewise in \((iii) q \) is the proposition that water is \( \text{H}_2\text{O} \). In the cases of \((v) q \) is the proposition that the continuum hypothesis is true. For \((i) q \) is more complex; it is the conjunctive proposition containing the chemical information concerning the structure of salt, the nature of dissolving, and the role of Coulomb’s law in both. If this is correct, then it appears that we can salvage the proposal that knowledge of necessity is fundamentally a priori.

Likewise even if \((S)\) is not a priori, it may remain the case that some a priori principle can, in the cases we have been considering, connect the proposition \( p \) to \( \square p \). Perhaps we should supplement the opening condition of \((S)\) with further chemical knowledge, in particular concerning the role that protons (but not neutrons) play in explaining the chemical behaviour of elements. Similarly, while we may not know either \((K1)\) or
(K2) *a priori*, it may be claimed that the disjunction of (K1) and (K2) is *a priori*.

However, we should not leap to the conclusion that there will always be some \( q \) filling this role, let alone that it should be easy to identify such a \( q \). The full version of the argument in the chemical case involves an argument from within chemistry. One might think that one could encapsulate the chemical information involved in a long conjunction that will perform the role of \( q \). However, it is highly questionable whether every *a posteriori* argument can be recast as an *a priori* argument with *a posteriori* premises. Furthermore, it is plausible that there are cases where the structure is similar to that in the dissolving salt case, but which are unknowable because of our human epistemic limitations.

I conclude, therefore, that if Kripke and Putnam are right about their examples, then there is reason to suppose that there are other cases that loosen the link between necessity and aprioricity even further.

Salmon does put forward a general argument to the effect that we cannot know necessity *a posteriori*. Necessary truths concern all possible worlds, whereas *a posteriori* investigation tells us only about the actual world. We can see that Tom is a cat, but since we see him only in the actual world, that method of generating knowledge cannot give us knowledge that he is a cat in all possible worlds. There is something suspicious about such an argument. *A priori* knowledge can be thought of as a limiting case of knowledge where the component of experience has been reduced to zero; if the component of experience is not reduced to zero, the knowledge is *a posteriori*. Thus *a posteriori* knowledge benefits from a greater range of epistemic resource than *a priori* knowledge. Nevertheless Salmon’s argument yields the conclusion that *a priori* thought can generate a greater range of knowledge than *a posteriori* reasoning, since the former can deliver knowledge of other possible worlds whereas the latter cannot. It seems odd that the more restricted form of thought should be capable of producing the greater output. To make sense of this I propose that it is implicit in Salmon’s argument that any *a posteriori* route to knowledge can be factored into two elements, an *a priori* element and a residual *a posteriori* element, where the latter is a matter of ‘pure experience’. It is the latter that delivers knowledge of this world only. But is there anything that fits this bill? The idea of pure experience, unadulterated by the workings of the intellect is an empiricist fiction.7 The thesis of the theory ladenness of observation tells us that we cannot factor out an *a priori* element from *a posteriori* knowledge leaving an element that is just raw sensation or experience, since all such experience is imbued with background information and involves the application of concepts.

2.

The Kripke–Putnam view of *a posteriori* necessities has been attacked from various quarters. Some of those attacks have cast doubt on whether in the relevant cases \( p \) is true at all; e.g. it has been doubted whether water is \( \text{H}_2\text{O} \) (e.g. on the grounds that sea-water is water but is not only \( \text{H}_2\text{O} \) or on the grounds that ice is \( \text{H}_2\text{O} \) but is not water). But such objections only partially weaken the conclusions reached. For, although in such cases, if established, we must delete \( \Box p \), the (*a priori*) argument for \( (p \rightarrow \Box p) \) still holds, where \( p \) is a proposition whose truth-
value is knowable only *a posteriori*. Further, what these critics do not notice is that they succeed in establishing that (A) holds for a different proposition, the denial that water is H\textsubscript{2}O. If we take ‘water is H\textsubscript{2}O’ to assert that water and H\textsubscript{2}O are one and the same substance, then their counterexamples establish that this is false. Kripke’s argument can be exploited to show that this is necessarily false. Thus we have:

\[(A^*) \Box \neg p; \neg L\neg p; L(\neg p \rightarrow \Box \neg p),\]

where \(p\) is the proposition that water is H\textsubscript{2}O. \((A^*)\) is of course just an instance of \((A)\), and other such instances can be generated for false propositions that are the formal analogues of \((a)-(d)\).

So, objections concerning the truth of the propositions employed as exemplifications of \((A)\) do not undermine the important philosophical lessons of the Kripke–Putnam arguments. Clearly, more significant than the actual truth of the propositions are the arguments that establish the truth of the material implications from such propositions to their necessitations. As mentioned, much of the focus has concerned whether those arguments proceed from purely semantic considerations or whether they import hidden essentialist assumptions. That is clearly an important discussion, but its outcome does little, on its own, to undermine the claim that there are propositions for which \((A)\) holds. Imagine that Nathan Salmon is right (as I am inclined to think), that Putnam does not establish the necessity of \((d)\) on semantic grounds alone – instead, one innocuous-looking assumption of his argument is already imbued with essentialism. That could (and, I am inclined to think, should) be taken to show just how intuitive essentialism is.

3.

I do not claim to have shown that *a posteriori* knowledge of necessity cannot be factored into an *a posteriori* component concerning non-modal facts and an *a priori* component concerning modal facts. That remains a plausible hypothesis. However, while possible I do not regard it as being in any way definitive, and the considerations given above suggest that this hypothesis stands in need of stronger confirmation than it has been given hitherto. That we can have unfactorizable *a posteriori* knowledge of modal truths remains an open epistemic possibility.

### Notes


3. The interpretation of Gödel’s first incompleteness theorem has been contested for precisely the reason that it seems to invite a rejection of the claim that the necessary truths of mathematics are all in principle knowable *a priori*. 

5 To be precise, Donnellan says concerning the principle that two liquid samples are consubstantial only if they have the same chemical structure, ‘I am inclined to think that we believe [it] on a posteriori grounds or, at least, not merely by consulting our linguistic skills. It strikes me as either being the product of scientific discovery, of scientific theories, or, perhaps, change in scientific outlook.’ Quoted in Salmon *Reference and Essence* p.165.

6 For this reason, the argument is sound even if \( w \) is in fact not genuinely possible.

7 See e.g. «A World of Pure Experience» William James (1904), *Journal of Philosophy, Psychology, and Scientific Methods*, 1, 533-543, 561-570.